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Periodic Response and Nonlinear Vibration Behavior for Automotive Clutch

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Modified HBM (Harmonic Balance Method) with AFT (Alternating Frequency Time) method is utilized to obtain steady-state response of an automotive clutch system with piecewise-linear stiffness. The stability analysis for the obtained response is performed via a perturbation technique and Floquet multipliers. The considered system shows a flip and fold bifurcation, and variation of system parameters can exhibit abnormal clutch vibration such as a rattling phenomenon.

Key Words: Rattle, Bifurcation, Stability, Perturbation, Discrete Fourier Transforms, Automotive Clutch

Nomenclature

C	: Viscous damping
h_1, h_2	: Non-dimension 1st, 2nd stage angle of drive side
I	: Equivalent mass moment of inertia of I_1 and I_2
I_1	: Equivalent mass moment of inertia of flywheel, clutch cover, crank shaft, connecting rod
I_2	: Equivalent mass moment of inertia of input gear and clutch hub
K	: Torsional spring stiffness of K_1 and K_2
K_1	: Torsional spring stiffness of clutch pre damper
K_2	: Torsional spring stiffness of clutch main damper
q	: Diffrence angle between θ_1 and θ_2
q_1, q_2	: 1st , 2nd stage angle of drive line
Q_n	: Harmonic component
r	: Crank radius
t	: Time
T	: Combined torque per one cylinder
T_C	: Torsional torque of clutch
T_C*	: Nondimensional torsional torque of clutch
T_E	: Engine fluctuation
y	: Nondimensional displacement
α	: Nondimensional viscous damping
β, β_2	: Nondimensional equivalent stiffness
δ	: Nondimensional gap
ζ	: Nondimensional damping
η, η_1	: Nondimensional frequency
θ	: Nondimensional time
θ_1, θ_2	: Angle
λ	: Eigenvalue
ν	: Subharmonic ratio
σ	: Stiffness ratio
ω, ω_1	: Angular velocity

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1. Introduction

Recent automotive power train system requires more compact design and higher efficiency as well as better ride comfortness. To satisfy above objectives the detailed clutch system analysis involving the characteristics of piecewise-linear nonlinearity is important. The inherent nonlinear system has shown non-regular type vibration and noise such as rattling and hammering phenomena, which can not be predicted or analyzed by linear vibration theory.

The hardening type spring characteristics has been widely known as a main cause of the abnormal clutch vibration (Ohnum, et al. , 1985). Generally the clutch vibration cannot be confined into the clutch system itself. This abnormal clutch vibration such as a rattle can cause driveline impact and whole automotive vibration since it is transmitted as an external input through gearbox's bearing (Shahin, 1984 ; Hedges, et al. , 1979).

A torsional character of a clutch can be represented by a hardening type or piecewise-linear type spring element (Kim, Noah, 1996). The important characteristics of this nonlinear system can be summarized as i) jump phenomenon, ii) abnormal vibration such as subsynchronous or supersynchronous vibration, iii) aperiodic vibration, and iv) chaotic vibration. All these nonlinear responses might have close relationships with rattling and hammering characteristics of an automotive clutch.

An experimental analysis of a piecewise-linear type clutch was performed and reported by Verschoore, (1991). He combined all the power train components' models, which were obtained by experimental works, and used numerical integration to analyze the nonlinear system behavior. However, he could not perform stability analysis, which might explain the cause of abnormal power train's vibration. Meanwhile, Stuhler used clutch and universal joint model in order to study resonance and instability problem (Stuhler, 1991). He also did not mention the stability characteristics for the system. Pardon et al. (1979) developed a software with which vibration and noise prediction of a drive system is possible. They also obtained the optimization condition for the given drive system using Monte Carlo method. However, they also have drawbacks not accounting for nonlinear characteristics. Padmanabhan et al. (1992, 1995) used a FPA(Fixed Point Algorithm) method to locate the possible solution for a piecewise-linear automotive clutch model and performed a stability analysis for the given system. Although their research is quite encouraging in this area, a FPA method they utilized inherently involves an accuracy problem unless adequate interpolation method is chosen (Kim, 1996).

In this paper, HBM(Harmonic Balance Method) with AFT(Alternating Frequency Time) approach is used for the analysis of a piecewise-linear automotive clutch system and the corresponding stability analysis will be performed to predict the possibility of abnormal vibration by changing various clutch parameters such as stiffness ratio, gap ratio, damping and frequency ratio.

2. Formulation and System Equation

The clutch model is shown in Fig. 1, and equations of motion for the given system are represented as follows:

$$\begin{aligned} I_1\ddot{\theta}_1 &= T_E\sin(\omega t) - C(\dot{\theta}_1-\dot{\theta}_2) - T_C(q) \\ I_2\ddot{\theta}_2 &= C(\dot{\theta}_1-\dot{\theta}_2) + T_C(q), \end{aligned} \tag{1.a}$$

where clutch torque is expressed as follows:

$$T_C(q) = \begin{cases} K_2 q + (K_1-K_2)q_1, & -q_2<q<-q_1 \\ K_1 q, & -q_1\le q\le q_1 \\ K_2 q - (K_1-K_2)q_1, & q_1<q<q_2. \end{cases} \tag{1. b}$$

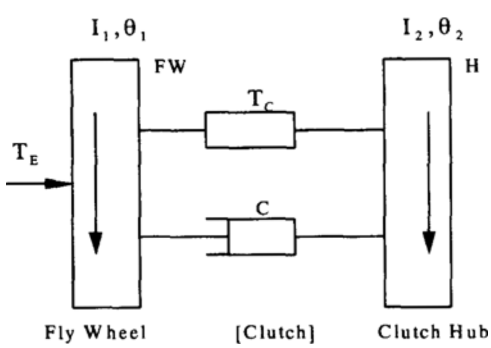


Fig. 1 The physical model.

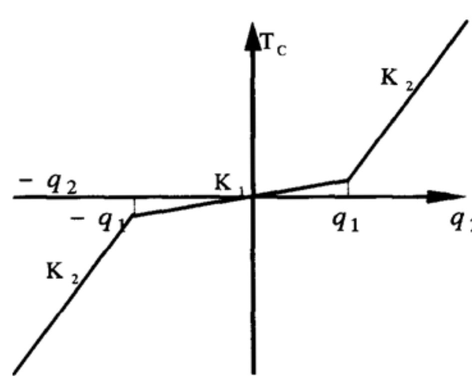


Fig. 2 Symmetric piecewise-linear torque.

In which, θ_1 is the angular displacement of flywheel, and θ_2 is the angular displacement of input gear. Also a superscript prime denotes differentiation with respect to the time t.

To study parameter effects for a given clutch system systematically, the following non-dimensional parameters can be introduced. The non-dimensional time (θ) is selected as a new independent variable as follows:

$$\nu\theta = \omega t, \tag{2}$$

where ν is a unit value for calculating harmonic vibration or super-harmonic vibration. Sub-harmonic vibration period will be ν times of excitation.

One can define the equivalent spring stiffness of K_1, K_2 as follows:

$$K = \frac{4K_1K_2}{(\sqrt{K_1}+\sqrt{K_2})^2}, \tag{3. a}$$

Also one can define the equivalent moment of Inertia of I_1, I_2 as follows:

$$I = \frac{I_1I_2}{I_1+I_2} \tag{3. b}$$

And equation (1) can be normalized as follows:

$$\ddot{y} + a\dot{y} + \nu^2 T_C{}^*(y) = \frac{\nu^2}{\eta_1{}^2}\sin(\nu\theta). \tag{4}$$

In above equations, all the definitions can be found in Appendix.

Figure 3 shows a physical model of the clutch system with the equivalent 1 degree of freedom. The steady-state response of equation (4) can be obtained using Fourier series as follows:

$$y(\theta) = a_0 + \sum_{n=1}^{k}(a_n\cos n\theta - b_n\sin n\theta), \tag{5}$$

where k represents the maximum harmonic term used to obtain steady-state solution. In the same fashion, nonlinear clutch torsional torque can be represented as follows:

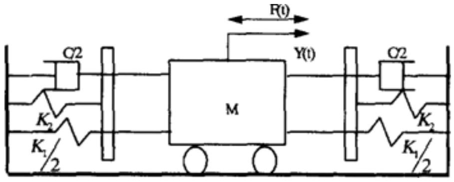


Fig. 3 Equivalent single degree of freedom model.

$$T_C^\circ(\theta) = c_0 + \sum_{n=1}^{k}(c_n\cos n\theta - d_n\sin n\theta), \tag{6}$$

In the above Eqs. (5) and (6), each coefficient is an unknown term to be determined. As the torsional torque is induced from nonlinear stiffness expressed as in appendix Eq. (A1), the coefficients of c_0, c_1, d_1, $\cdots$, c_n, d_n are the function of a_0, a_1, b_1, $\cdots$, a_n, b_n. By substituting Eqs. (5) and (6) into Eq. (4) and rearranging all the trigonometric terms, the following Eq. (7) can be obtained by the following matrix form.

$$[A]\{X\} = \{L\} + \{g\} \tag{7}$$

in which

$$[A] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & a & 0 & 0 & \cdots & 0 & 0 \\ 0 & a & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 4 & 2a & \cdots & 0 & 0 \\ 0 & 0 & 0 & 2a & -4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & k^2 & ka \\ 0 & 0 & 0 & 0 & 0 & \cdots & ka & -k^2 \end{bmatrix} \tag{7. a}$$

$$\{X\} = \{a_0 a_1 b_1 a_2 b_2 \cdots a_k a_k\}^T \tag{7. b}$$

$$\{f\} = \{\nu^2c_0\,\nu^2c_1 - \nu^2d_1\,\nu^2c_2 - \nu^2d_2\cdots\nu^2c_k - \nu^2d_k\}^T \tag{7. c}$$

$$\{g\} = \left\{0\ 0\ -\phi\frac{\nu^2}{\eta_1{}^2}0 - \phi\frac{\eta_1{}^2}{\nu^2}\cdots 0 - \phi\frac{\eta_1{}^2}{\nu^2}\right\}^T \tag{7. d}$$

$$\phi = \begin{cases} 1, & \nu = n \\ 0, & \nu \neq n \end{cases} \quad n = 1, 2, \cdots, k \tag{7. e}$$

As Eq. (6) is represented by the nonlinear terms, the Eq. (7) is automatically a nonlinear one. As the unknown coefficients of Eqs. (5) and (6) are coupled with the Eq. (A1), both coefficients can be determined by using AFT technique. In order to apply AFT technique, the given harmonic components of the discrete displacement P_r with r'th discrete time can be obtained using IDFT (Inverse Discrete Fourier Transform) as:

$$P_r = \mathrm{Real}\left\{\sum_{n=0}^{k} Q_n\left(\cos\frac{2^5 nr}{N} + i\sin\frac{2^5 nr}{N}\right)\right\};$$
$$r = 0, 1, \cdots, N-1 \tag{8}$$

where Q_n is the nth harmonic component, i.e., $a_n + ib_n$.

Nonlinear restoring torque term of the Eq. (A2) can be represented in discrete time as

$$s_r=\begin{cases}\beta_2 P_r+\delta, & P_r<-h\\ \beta_1 P_r, & -h\le P_r\le h\\ \beta_2 P_r-\delta, & P_r>h\end{cases} \tag{9}$$

The corresponding restoring torque in frequency domain can be obtained using DFT (Discrete Fourier Transform) as follows:

$$R_n=\frac{\Psi}{N}\sum_{n=0}^{\infty}s_r e^{i\left(-\frac{2\pi nr}{N}\right)};\qquad \Psi=\begin{cases}1, & n=0\\ 2, & n\neq 0\end{cases} \tag{10}$$

where R_n represents for the nth harmonic components, i.e. c_n+id_n.

As Eq. (7) is a nonlinear algebraic one, the incremental unknown coefficients of ΔX can be written using *Newton-Raphson method as*

$$[J]\Delta X+G=0, \tag{11}$$

where $J=\left[\frac{\partial G}{\partial X}\right]$ is a Jacobian matrix, and vector G is obtained from Eq. (8). Where G is a vector with the dimension of $(2N+1)\times 1$.

3. Stability Analysis

One of important advantages of HBM with AFT is its ability to accommodate the criteria for bifurcation and stability conditions. To analyze the given nonlinear system response systematically, one should determine the stability of the given periodic solution. This stability analysis is quite important in clutch system to study the parameter variation, which sometimes involves sudden vibration changes such as jumping, subsyncronous, and supersyncronous vibration. To determine the stability information for the obtained periodic solution, eigenvalues of a monodromy matrix can be utilized. The stability analysis is based on the perturbation of obtained periodic solution. The perturbated equation for periodic response can be written as:

$$\Delta\ddot{y}+\alpha\Delta\dot{y}+\nu^2\beta(r)\Delta y=0, \tag{12}$$

where $\beta(r)$ is a discontinuous function whose value depends on the $r'th$ discrete time.

Equation (12) can be rewritten using the 1st order form as:

$$[\dot{Z}]=[u(\nu\theta)][Z],\ [Z(0)]=[I]. \tag{13}$$

Eigen values of λ_1, λ_2 for monodromy matrix $[Z(\nu\theta)]$ become Floquet multipliers, and have the following relation

$$\lambda_1\lambda_2=e^{-2\alpha T} \tag{14}$$

where T represents one period ($T=2^5$). As the system parameter varies, the obtained response can be unstable as one of Floquet multipliers can leave the unit circle through three possible routes.

4. Numerical Results and Discussion

The steady-state response based on HBM, is compared with that of the numerical integration in Fig. 4. HBM could give an accurate steady-state response efficiently compared with analytical or numerical methods.

In this paper, the number of retained harmonic terms is chosen to be four considering computational time and accuracy. The steady-state response obtained by HBM was analyzed

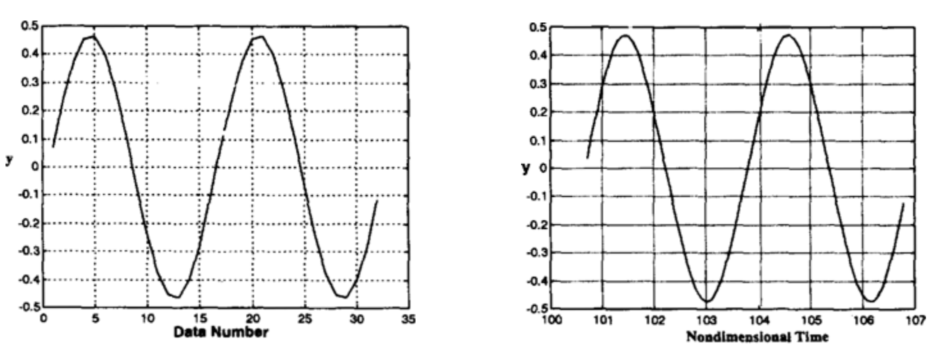


Fig. 4 Comparison HBM with Runge-Kutta. ($\sigma=10$, $\zeta=01$, $\eta=1.5$, $\eta_1=2.0$, $\nu=2$, $H=0.1$, $\delta=0.15$).

by discrete time method on data number needed to response of 2π period, which means $\nu=1$.

Steady-state response of 2^5 period for Runge-Kutta method is identical to the result of HBM. By changing various system parameters, for the given stable solution of $\nu\theta$ period.

One of the most interesting situation in piecewise-linear system is that it can sometimes have "jump phenomenon" (Stuhler, 1991). As the clutch system considered here has strong nonlinear characteristic, jump phenomenon, which can be verified by the fact that one of Floquet multipliers leaves the unit circle via $+1$, might be possible and the result is plotted in Fig. 5.

This is called fold bifurcation or saddle-node bifurcation. Figure 5 shows three different responses depend on the initial conditions. In this simulation, the system shows unstable and violent vibration. In Fig. 6, the flip bifurcation is found when one of Floquet multiplier leaves unit circle through -1. Figure 6 shows non-dimensional displacement (y) versus non-dimensional time (θ) for the flip bifurcation case, and it shows unusual vibration. In the frequency domain, dominant peak at 1/3 of the forcing frequency appears in case of the flip bifurcation. These phenomena can make unstable vibration in automotive clutch.

Fig. 7 represents stable-unstable curve of non-dimensional frequency (η) versus non-dimensional damping (ζ) change based on HBM analyses. We can find more stable solution as we increase the damping (ζ) while keeping the frequency constant.

5. Conclusion

HBM with AFT technique is adopted to obtain accurate steady-state responses of the nonlinear automotive clutch with a single excitation torque. For the numerical computation of clutch

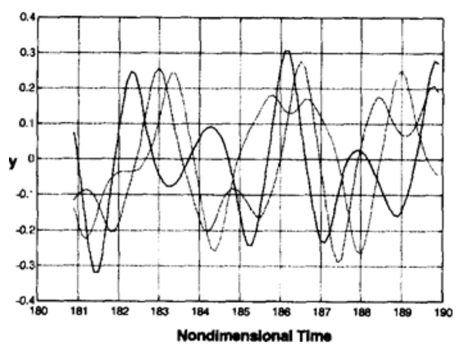


Fig. 5 Response depending on initial condition; solid line: 0.1, dashdot line: 0.2, dot line: 0.3 ($\sigma=10$, $\zeta=0.15$, $\eta=1.2$, $\eta_1=2.0$, $\nu=2$, $H=0.08$, $\delta=0.18$)

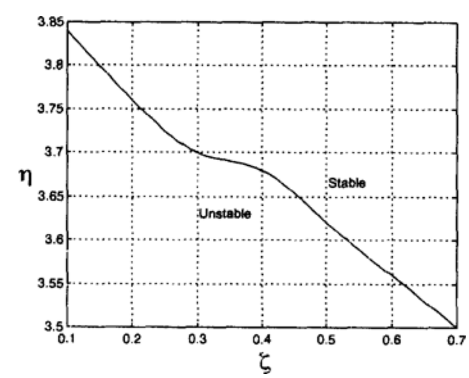
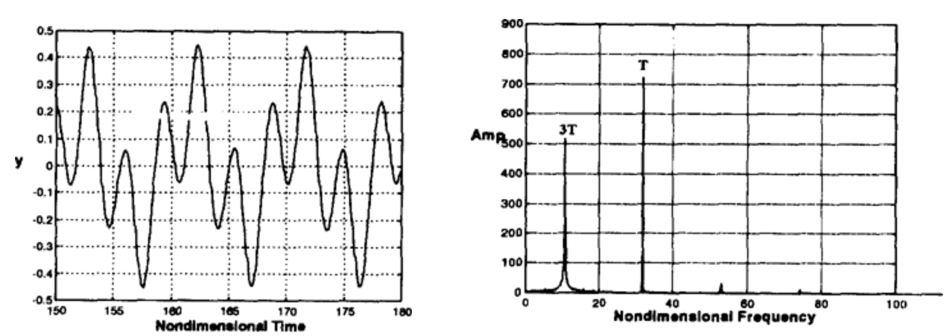


Fig. 7 Bifurcation on $\zeta-\eta$ graph ($\sigma=10$, $\nu=2$, $H=0.1$, $\delta=0.15$)



Time history

FFT result

Fig. 6 Flip bifurcation $[T\rightarrow 3T]$ ($\sigma=10$, $\zeta=0.6$, $\eta=2.7$, $\eta_1=2.0$, $\nu=2$, $H=0.1$, $\delta=0.15$).

response, four discrete harmonic terms were utilized considering numerical efficiency as well as computational accuracy.

Stability of the obtained response is analyzed using bifurcation approach. Response based on HBM is compared with that from numerical integration with Runge-Kutta method. Two results show good agreement. The system shows flip and fold bifurcation, which have a strong relationship with the clutch rattle. One could conclude that HBM is advantageous in predicting unstable clutch vibration through its stability criteria.

This method can be applied to the clutch system with hysterisis. More generally, This method can be extended to study multi-degree of freedom and multi-dimensional nonlinear system with nonlinear vibration phenomena.

References

Hedges, J.L and Buttler, K.J., 1979, "A CAD System for the Anlysis of Vehicle Drivline Noise," *ImechE,* pp. 121～159.

Kim, Y. B., Noah, S. T. , 1996, "Quasi-periodic Response and Stability Analysis for Nonlinear Jeffcott Rotor," *Journal of Sound & Vibration,* Vol. 190, pp. 239～253.

Kim, Y. B., 1996, "Quasi-periodic Response and Stability Analysis for Non-linear Systems: A General Approach," *Journal of Sound and Vibration,* Vol. 192. pp.

Ohnum, S. ohnum, S. and Yahata, S., 1985, "Research on Idling Rattle of Manual Transmission," *SAE* 850979, pp. 159～167.

Padmanabhan, C. and Singh, R., 1995, "Dynamics of a Piecewise Non-linear System Subject to Dual Harmonic Excitation using Parametric Continuation," *Journal of Sound and Vibration,* Vol. 184, pp. 767～799.

Padmanabhan, C. and Singh, R., 1992, "Spectral Coupling Issues in a Two-degrees-of-freedom System with Clearance Non-linearities," *Journal of Sound and Vibration,* Vol. 155, pp. 209～230.

Padoan, R. et al., 1979, "Industrial Vehicle Drive Line Design and Optimization for Low Noise and Vibration Levels," *ImechE,* pp. 149～159.

Shahin, M. A., 1984, "A Neumerical Technique for Theoretical Analysis of Transmission Systems," *Vehicle Noise and Vibration,* pp. 107～112.

Stuhler, W. , 1991, "Dynamic Behavior of a Driverline with Universal Joints," *IPC,* pp. 329～338.

Verschoore, R., 1991, "Vehicle Drive Line Dynamic Behavior Simulation," *IPC*, pp. 321～327.

Appendis

All definitions used in Eq. (4) are as follows:

$$T_c^*(y)=\begin{cases}\beta_2 y+\delta & -h_2<y<-h_1\\ \beta_1 y & -h_1\leq y\leq -h_1\\ \beta_2 y-\delta & -h_1<y<-h_2\end{cases} \tag{A1}$$

$$y=\frac{K}{T_E}q=\theta_1-\theta_2 \tag{A2}$$

$$\omega_n=\sqrt{\frac{K}{I}},\quad \omega_1=\sqrt{\frac{K}{I_1}} \tag{A3}$$

$$\eta=\frac{\omega}{\omega_n},\ \eta_1=\frac{\omega}{\omega_1} \tag{A4}$$

$$\zeta=\frac{C}{2I\omega},\quad \alpha=\frac{2\nu\zeta}{\eta} \tag{A5}$$

$$\delta=\frac{(K_2-K_1)\,q_1}{\eta^2 T_E} \tag{A6}$$

$$h_1=\frac{K}{T_E}q_1,\ h_2=\frac{K}{T_E}q_2 \tag{A7}$$

$$\sigma=\frac{K_2}{K_1} \tag{A8}$$

$$\beta_1=\left(\frac{1+\sqrt{\sigma}}{2\eta\sqrt{\sigma}}\right)^2,\ \beta_2=\left(\frac{1+\sqrt{\sigma}}{2\eta}\right)^2 \tag{A9}$$

in which the dot indicates a differentiation with respect to non-dimensional time τ.